



## Identification of High School Students' Thinking Process Levels in Solving Geometry Problems Using Van Hiele's Theory

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### Abstract

This research aims to identify level of students' thinking process in solving geometry problems based on Van Hiele's theory. The Van Hiele's theory is the theory of geometry thinking process, consist of level 0 visualization, level 1 analysis, level 2 informal de-duction, level 3 deduction, and level 4 rigor. The qualitative approach was used in this research. The participants were four voluntary students come from the second grade of Senior High School. Data collected by observation, test, and interview. The re-sults show that during the problem-solving process, the male students thinking process is at an analysis level or level 1 and female students are at an informal deduction level or level 2. The result of the research especially deals with students' thinking level can be used as teachers information in preparing teaching strategies that appropriate with their students' thinking especially in geometry. Furthermore, the result can be used to minimize other factors that inhibit students' thinking process in solving geometry problems.

**Keywords:** geometry; problem solving; thinking process.

## INTRODUCTION

Thinking and problem solving are one. Thinking is a variety of activities that use concepts and symbols instead of objects and events, while the directed thinking process is a predetermined thinking process that is directed at something, usually directed at solving a problem (Maulidya, 2018). Solving a mathematical problem is known as mathematical problem-solving ability where this ability is the core of mathematics learning itself and one of the higher-level thinking skills (Hendriana, Rohaeti, & Sumarmo, 2017, Hidayat & Sariningsih, 2018). It means problem solving in mathematics education related to mathematical problem.

Mathematics is the science of logic to understand shapes, structures, quantities and other concepts related to large numbers and is divided into 4 branches, namely arithmetic, algebra, geometry and analysis (Utomo, 2015). Learning mathematics means using language that is based on rules or rules that must be learned so that motivating students to be able to express objects into mathematical language is very important (Schoenfeld, 2016).

According to Hamzeh (2017) geometry is the true link between mathematical concept and the real world, so that there is a geometry problem. Capabilities in solve problems in the real world by using the skills, strategies and measures and knowledge they have to apply in relevant new problem situations (Rosmita, 2020). It is related to Saunders, Spooner & Ley Davis (2018), mathematical concepts especially geometry concept and problem solving are parts of human life used in daily life and required for being able to navigate the world such as creating budgets, determining distances, determining temperatures, and understanding time.

Beside that, in studying mathematics, especially geometry, it is necessary to pay attention to various factors including willingness, intelligence, ability, presentation methods, readiness of teachers and students, in this case the gender difference of students, namely boys

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and girls, as well as the curriculum used (Budiarti & Mahendra, 2020). Gender differences can result in differences in student learning psychology, so male and female students certainly have many differences in learning mathematics (Nugraha & Pujiastuti, 2019, Budiarti & Mahendra, 2020). In addition, one of the factors that affects problem solving is gender (Afrida, Hamam & Akbar, 2019). This shows that there is a link between mathematical problem solving and student gender.

Van Hiele said that geometry learning depends on students' level of thinking consists of level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), level 4 (rigor) and teachers play an important role in facilitating students to move to the next level by providing an adequate learning experience (Pertiwi, & Sudihartinih, 2020). According to Siregar (2016), problem solving capabilities consists of 1) Understanding the problem; (2) devising plan; (3) carrying out the plan; (4) Looking Back. Schoenfeld (1985) developing the heuristic process from Polya called five episodes or Schoenfeld's model that were consists of (1) analysis; (2) design; (3) exploration; (4) implementation (5) verification and the heart of heuristic process is exploration. According to Mujib (2016), Schoenfeld's theory can be used as a more appropriate mathematical problem-solving process to be applied in schools.

The thinking process in solving geometry problems based on Van Hiele's theory conducted by Bada (2019) shows that male students tend to use visualization thinking processes (level 0) in the analysis and design problem solving episodes and female students tend to use analysis thinking processes (level 1) in the analysis and design problem solving episodes. Furthermore, Khaerunnisa's (2023) research found that 16% of students were at level 0 (visualization), 43% of students at level 1 (analysis), 30% of students at level 2 (informal deduction), 11% of students at level 3 (formal deduction), and none of the students managed to reach level 4 (rigor). According to Nurani, Irawan & Sa'dijah (2016), the student reached judging from the gender, female students whose high ability reached level 1 (analysis) while low ability female students were at level 0 (visualization). High ability male students were at level 1 (analysis) while male students whose low ability was at level 0 (visualization). This shows that the students' thinking process has not increased significantly. This study aims to identify level of students' thinking process in solving geometry problems.

## **METHODS**

This research was conducted on February 2024 in one of the Senior High School in Sikka. The participants were second grade students and selected based on voluntariness in accordance with the ethical issues in educational researching. They were two male students called S1, S3 and two female students called S2, S4. According to the aims of this study, the theories used in this research were heuristic process theory initiated by Schoenfeld which is related to Van Hiele's theory as served in Table 1.

Qualitative approach was used in this research. This helped to analyze the data that were took by test, interview and observation. Test and interview were arranged based on problem solving episodes by Schoenfeld 1985. The geometry problem solving test consist of two problems called Problem 1 and Problem 2 adopted from As'ari, Tohir, Valentino, Imron, & Taufiq (2016). Test and interview were given in sequence. After the participant finished the test, they were interviewed. Observation did during test time. After data collected, validating data used method triangulation.

Table 1. Heuristic Process Theory Initiated By Schoenfeld Which Is Related To Van Hiele's Theory

Heuristic Process	Van Hiele's Theory
<p><b>Analysis Episode</b></p> <ul style="list-style-type: none"> <li>- Student can determine sufficient terms and necessary terms</li> <li>- Student can reformulating the problem in other ways</li> </ul>	<p><b>Visualizaion Level (Level 0)</b></p> <ul style="list-style-type: none"> <li>- Students can recognize geometric forms (shape/ solid) based on visual characteristics and appearance of objects or looking at objects as a whole</li> <li>- Students can mention the geometric concepts used to solve problems</li> </ul>
<p><b>Design Episode</b></p> <ul style="list-style-type: none"> <li>- Student can explain the reason of choosing the mathematical concept to solve the problem</li> </ul>	<p><b>Analysis Level (Level 1)</b></p> <ul style="list-style-type: none"> <li>- Students can determine the characteristics of geometric forms by observing, measuring, experimenting, drawing and modeling</li> <li>- Students can determine the properties of the geometry concept used in problem solving by observing, measuring, experimenting, drawing and modeling</li> </ul>
<p><b>Exploration Episode</b></p> <ul style="list-style-type: none"> <li>- Essentially equivalent problems means student can determine the mathematical concept and procedures of problem solving imitating the concept and procedures have been used</li> <li>- Slightly modified problems if student can explain the relationship between mathematical concept and procedures related to solve the problem</li> <li>- Broadly modified problem if student can explain the procedures of problem solving using the mathematical concept have been learn.</li> </ul>	<p><b>Informal Deduction Level (Level 2)</b></p> <ul style="list-style-type: none"> <li>- Students determine the relationship between characteristics of one geometric form and characteristics of some geometric forms</li> <li>- Students determine the relationship of a geometric concept and some geometric concepts</li> <li>- Students determine the characteristics of various geometric forms</li> <li>- Students determine various geometric concepts</li> </ul>
<p><b>Implementation Episode</b></p> <ul style="list-style-type: none"> <li>- Student can run and explain the steps described in analysis and exploration episodes</li> </ul>	<ul style="list-style-type: none"> <li>- Students classify geometric forms hierarchically</li> <li>- Students classify geometric concepts hierarchically</li> <li>- Students make abstract definitions</li> </ul>
<p><b>Verification Episode</b></p> <ul style="list-style-type: none"> <li>- Student can recheck her/ his answer in analysis, design, exploration dan implementation episodes</li> <li>- Student can recheck her/ his answer in analysis, design, exploration dan implementation episodes with anothe procedures or mathematical concept.</li> </ul>	<p><b>Deduction Level (level 3)</b></p> <ul style="list-style-type: none"> <li>- Students prepare proof of statement using geometric concepts that have been obtained</li> <li>- Students develop evidence in more than one way.</li> </ul>

**RESULTS AND DISCUSSION**

The results are shown in two parts, namely targeting the answers of male students and female students. The answers of male students can be seen on figure 1 and figure 2.

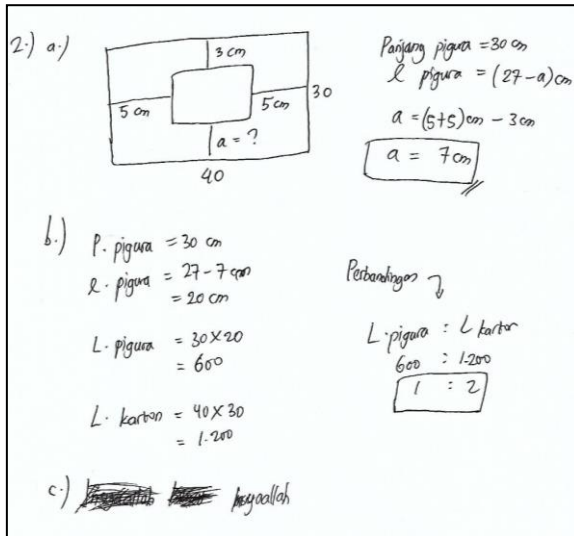


Figure 1. S1's Answer Sheet

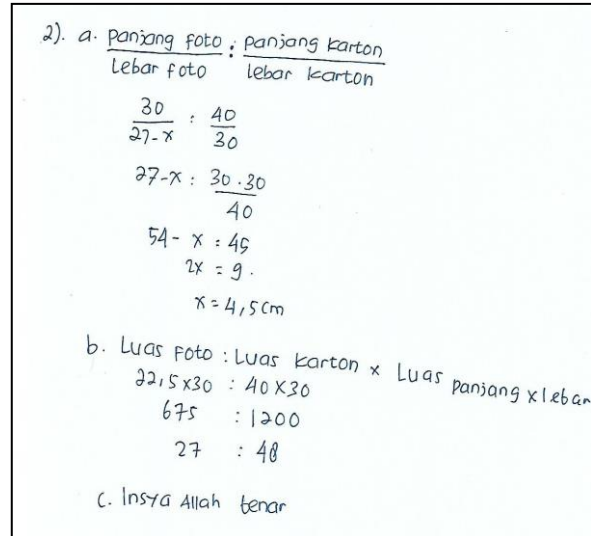


Figure 2. S3's Answer Sheet

The answers of male students based on the problem-solving episode are as follows at analysis and design episodes, students asked in interviewing and for problem 2, S1 can mention the concept used on problem 2 is rectangle and understanding the problem with try to drawing the problem on his answer sheet; while S3 can mention the concept used on problem 2 is rectangle, proportion and congruence, and understanding the statement because there is a figure of it. At analysis and design episodes in problem 1 and problem 2, male students can mention the concept used by recognizing the shapes given in the problems on visual characteristics and the appearance of the objects. It shows male students thinking tend at a visualization level (Nurani, Irawan & Sa'dijah, 2016, Alex & Mammen, 2016).

At exploration episode, students asked to answer the question a). S1 said not correct with the shapes properties especially the length of their sides; while S3 said correct with their properties especially look at the similarity of the shapes sides example a rectangular looks like a parallelogram. Beside that, for problem 2, students asked to answer the question a). S1 has a wrong answer. It is showed in determining the length and width from pigura and photo; while S3 has a right answer. S1 and S3 can determine the characteristics of geometric forms by observing, measuring or the can analyze and name the properties of shapes but can't create the meaningful of shapes properties; while in problem 2, S1 can recognize the figure but can't identify it's properties; while S3 can recognize the figure and identify it's properties. It's mean S1 and S3 thinking tend at an analysis level or level 1 (Nurani, Irawan & Sa'dijah, 2016 Alex & Mammen, 2016).

At implementation episode, students asked to answer question b). S1 and S3 sequence the toys according to their answer in question a). S1 sequence them from I, II, IV, III, V, and VI; while S3 sequence them from V, II, I, IV, VI and III. For problem 2, students asked to answer question b). S1 and S3 used their answer in question a) and the results are S1 still has a wrong answer; while S3 has a right answer. At implementation episode, in problem 1 and problem 2, male students still using their answer on exploration episode. It means they can not try to change their answer. So, they are still thinking tend at a same level in this episodes. They thinking tend at an analysis level or level 1.

At verification episode, students asked to answer question c). S1 try to answer if it's true, thank God, if it's wrong, it's learning; while S3 don't answer the question. In interviewing, S1 says he hope his answer is right; while S3 says he can not writes because he doubt with his answer. For problem 2, students asked to answer question c). S1 answer "Insyaallah"; while S3 answer "Insyaallah, right". In interviewing, S1 says he hope his answer

is right; while S3 says he believes with his answer. At verification episode, S1 can answer in problem 1 and problem 2; while S3 can answer in problem 2. They try to answer, but they can not give the reason to support they answer. It shows that they can answer but they can't create the meaningful of their answer. They thinking still tend at analysis level or level 1 (Nurani, Irawan & Sa'dijah, 2016, Alex & Mammen, 2016).

The answers of female students can be seen on figure 3 and figure 4. The answers of male students based on the problem-solving episode are as follows at analysis and design episodes, students asked in interviewing. S2 and S4 can mention the concept used on problem 1 is shape by recognize the shapes given in problem 1 and called the types of shapes from I – VI that are square, rhombus, parallelogram, rectangle, kite and isosceles trapezoid. Beside that, for problem 2, students asked in interviewing. S2 can mention the concept used on problem 2 is rectangle, proportion, congruence and she understands the statement in problem 2 because there is a figure of it; while S4 can mention the concept used on problem 2 is rectangle, proportion, congruence and she understands the statement in problem 2 with drawing the problem on her answer sheet and change the variable  $a$  to  $x$ . At analysis and design episodes, in problem 1, S2 and S4 can mention the concept used by recognizing the shapes based on visual characteristics. It shows female students thinking in problem 1 at a visualization level or level 1. But in problem 2, S2 can mentioned the concept used in this problem with correct answer. It shows she can, can determine the properties of the geometry concept used in problem solving by observing, experimenting. So she thinking's tend at an anaysis level or level 1 while S4 can mentioned the concept with right answer, and she can makes her own variable. It shows she thinking's tend at an informal deduction level or level 2 (Nurani, Irawan & Sa'dijah, 2016, Alex & Mammen, 2016).

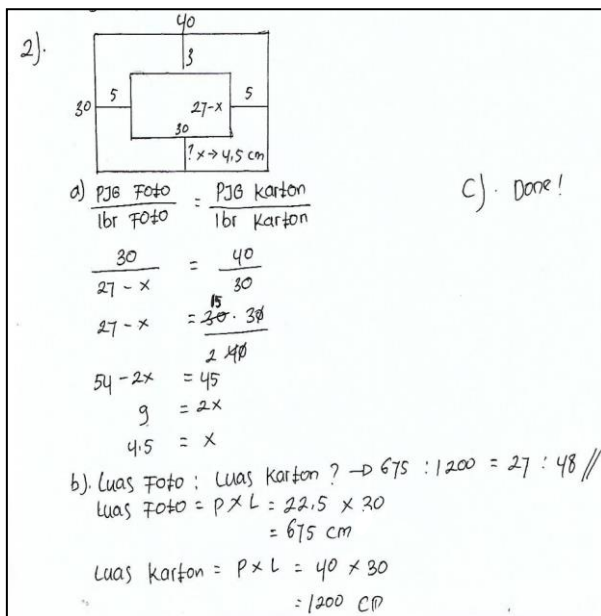


Figure 3. S2's Answer Sheet

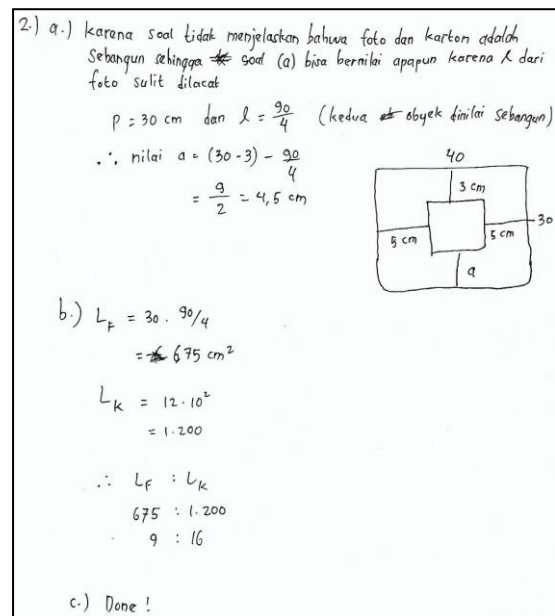


Figure 4. S4's Answer Sheet

At exploration episode, students asked to answer the question a). S2 said not correct with the shapes properties especially the length of their sides and their order of rotational symmetry; while S4 said correct with their properties especially the length of their sides, their order of rotational symmetry and their axes of symmetry. For problem 2, S2 and S4 has a right answer. At exploration episode, in problem 1 and problem 2, female students can determine and classify geometric forms and concepts hierarchically. It's shows S2 and S4

thinkings tend at an informal deductional level or level 2 (Nurani, Irawan & Sa'dijah, 2016, Alex & Mammen, 2016).

At implementation episode, students asked to answer question b). S2 and S4 sequence the toys according to their answer in question a). S2 sequence them from many to few of axes of symemetry that are I, II, III, IV, V, VI; while S4 sequence them opposite the S2's answer that is from VI, V, IV, III, II, I. For problem 2, students asked to answer question b). S2 and S4 used their answer in question a) and the results shows that they have a right answer. At implementation episode, in problem 1 and problem 2, female students still using their answer on exploration episode. It means they still thinkings tend at an informal deductional level or level 2.

At verification episode, students asked to answer question c). S2 and S4 have the same answer. They said "done". In interviewing, they answers they believed they answer is right because they memorize the concept of shape properties. For problem 2, S2 and S4 have the same answer. They said "done". In interviewing, they say that they ever finished the problem that similiarity with problem 2. At verification episode, in problem 1 and problem 2, female students can answer and they can create the meaningful of theirs answers with memorizing the concept of shape properties. It means they still thinkings tend at an informal deductional level or level 2 (Nurani, Irawan & Sa'dijah, 2016, Alex & Mammen, 2016).

During the problem-solving test, students sitting far apart to each other, so they can't cheating and can't try to help each other. They start the test in the same time after test and answer sheets given to them. Duration time of test is 90 minutes. They collect the answer sheet on the same time when the time up. During the problem-solving test, students are still in same condition. The results show that during the problem-solving process, the male students thinking process tend at an analysis level or level 1 and female students thinking tend at an informal deduction level or level 2. It is caused by learning geometri received by students. In interviewing, female students explaining that they ever solve the similiarity problem but the male students are not. It means, level thinking of solving geometry problems depend on learning geometry process and the subject of learning geometry process is teacher and students. Related to Muhassanah, Sujadi, & Riyadi, (2014) geometry learning depends on students' level of thinking and teachers play an important role in facilitating students to move to the next level by providing an adequate learning experience.

## **CONCLUSION**

Every episode of problem solving especially in geometry problems has a different thinking process between male and female students. At analysis and design episodes, male students thinking tend at a visualization level or level 0, while female students thinking tend able to at an analysis level or level 1. At exploration, implemenation and verification episodes, male students thinking tend at analysis level or level 1, while female students thinking able tend to at an informal deduction level or level 2. This result related to Van Hiele said about geometry thinking. The result of the research especially deals with students' thinking level can be used as teachers information in preparing teaching strategies that appropriate with their students' thinking especially in geometry. Furthermore, the result can be used to minimize other factors that inhibit students' thinking process in solving geometry problems.

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